

Final Report
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**IMPROVED DESIGN AND ANALYSIS
OF PRESTRESSED CONCRETE GIRDERS**

by

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ABSTRACT

A design program and an analysis program were written to take advantage of more exact analytical methods. Both programs employ a very accurate analytical model that uses the actual stress-strain relationships for concrete and prestressing steel. The model also includes the effect of additional prestressing steel strains that occur during live loading. In addition, the analysis program incorporates the fact that the strength properties of prestressed concrete bridge girders are not deterministic, but rather are random variables. Both programs can accept several standard cross-sections or any girder cross-section made up of a combination of rectangular and triangular areas.

The design program, named MXPSD (**More eXact PreStress Design**), designs for flexural stresses at **midspan**. The total number of strands (with a specified center of gravity) required to provide a specified level of stress in the precompressed tensile zone is found. Comparisons with existing bridge designs indicate that MXPSD produces significant savings in prestressing steel for some prestressed bridges. Since MXPSD usually predicts lower concrete stresses, it is expected that additional savings could be realized in terms of required concrete strength. Because of these potential savings, MXPSD should be quite useful to bridge design agencies.

The analysis program, named PROCAT (**PRObabilistic capacity Analysis Tool**), performs either a deterministic, mean-value analysis or a probabilistic analysis. The deterministic analysis produces a single mean-value cracking moment capacity; the probabilistic analysis produces a population of cracking moment capacities. Using this population and the statistical information about a live load, a user can find the probability of failure (by flexural cracking) associated with the passage of the load. This program

enables an assessment, in a much more rational way, of the safety of the passage of an extreme live load. The program should be extremely useful to agencies dealing with overload permits, load limits, or construction loads.

IMPLEMENTATION

All three programs developed for this project have been implemented in the Bridge Design Section of the Idaho Transportation Department. A users manual is being completed with the input of the ITD staff and a training session for Bridge Design personnel will be held. In addition, it is anticipated that the Principal Investigator will present the findings of this project at various professional conferences in the future.

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PROBLEM STATEMENT

As used both for bridge design and analysis, conventional methods to calculate moment capacity make many simplifying assumptions and are deterministic in nature. Present methods usually use the properties of the gross uncracked section and the assumption that the concrete stress-strain distribution is linear.

Design:

The conventional design approach ignores the additional prestressing steel strain (and therefore load capacity) caused by the application of the live load. For some time, the accepted method of analyzing cracked prestressed (and partially prestressed) girders has included the effect of these additional prestressing steel strains. By ignoring them, the designer may be unnecessarily conservative. The assumption that the concrete stress-strain distribution is linear is also conservative. The use of more exact techniques may lead to some economy in design.

Analysis:

Bridge engineers often need to determine the maximum safe load that may be allowed to pass over a prestressed bridge. Most bridges experience the passage of at least one near-critical load during their design life, often during construction. Many times the need for the load to pass is urgent, requiring a swift but accurate evaluation of safety. A conventional analysis uses deterministic strength properties, ignoring the random nature of such variables as concrete strength and prestress force. In controlled cases, the extreme load may reasonably be assumed to be of a deterministic value, but the moment capacity

is a random variable. Because of this, the conventional analysis does not reveal the probability of failure associated with the passage of the extreme load. Therefore, bridge engineers have been forced to try to account for the probability of failure by a combination of experience and intuition. This approach is often disquieting for those engineers required to set load limits, issue overload permits, or approve temporary construction loads. Using probabilistic methods, the analysis would calculate a population of moment capacities rather than a single deterministic value. This would give the bridge engineer a tool to assess, in a more rational way, the risk associated with the passage of an extreme load.

OBJECTIVE AND SCOPE

The more exact design and analysis methods described above are not typically used because of the long and difficult calculations required. The objectives of this study are as follows:

1. Develop a computer program to design prestressed concrete girders, taking advantage of more exact design assumptions.
2. Develop analysis software capable of finding either deterministic or probabilistic moment capacities.
3. Both the design and the analysis programs should be able to accept certain standard cross-sections or any girder cross-section made up of a combination of triangular and rectangular areas.
4. Write a computer program to build the data files used by the design and analysis programs.
5. Implement the software in the Idaho Transportation Department.

PROJECT DESCRIPTION

INTRODUCTION

The software consists of three programs: one for input (**STEP1**); one for design (MXPSD); and one for analysis (PROCAT). All three programs are based upon programs written by Geidl (1) for simulation of bulb-tee girders. The software allows the user to investigate any girder shape composed of triangles and rectangles. Several predefined, standard shapes are available to the user, and additional shapes can be easily added to the standard shape list. The analysis software accommodates batch or interactive runs and has been compiled and run on IBM 4300 series computers under **VM/CMS** and **MVS/XA2.2** operating systems. The original programs, which were the basis for this software, have also been run on VAX, Prime, and Cray computers, so good portability is expected. In addition all three programs have been compiled and run on IBM PC/AT and compatible microcomputers. All of the program modules are written in ANSI Fortran 77.

INPUT PROGRAM

The input program is named **STEP1**. This software prepares the necessary input files for both the design and the analysis programs. Some important features of the program are as follows:

1. The user has the option of preparing input for either design or analysis.
2. The user has the option of selecting one of several standard shapes or a nonstandard shape.
3. For nonstandard shapes, the user subdivides half of the cross section into rectangular and triangular areas; the program assumes the shape has a vertical axis of symmetry. An example section appears in Fig. 1. The maximum

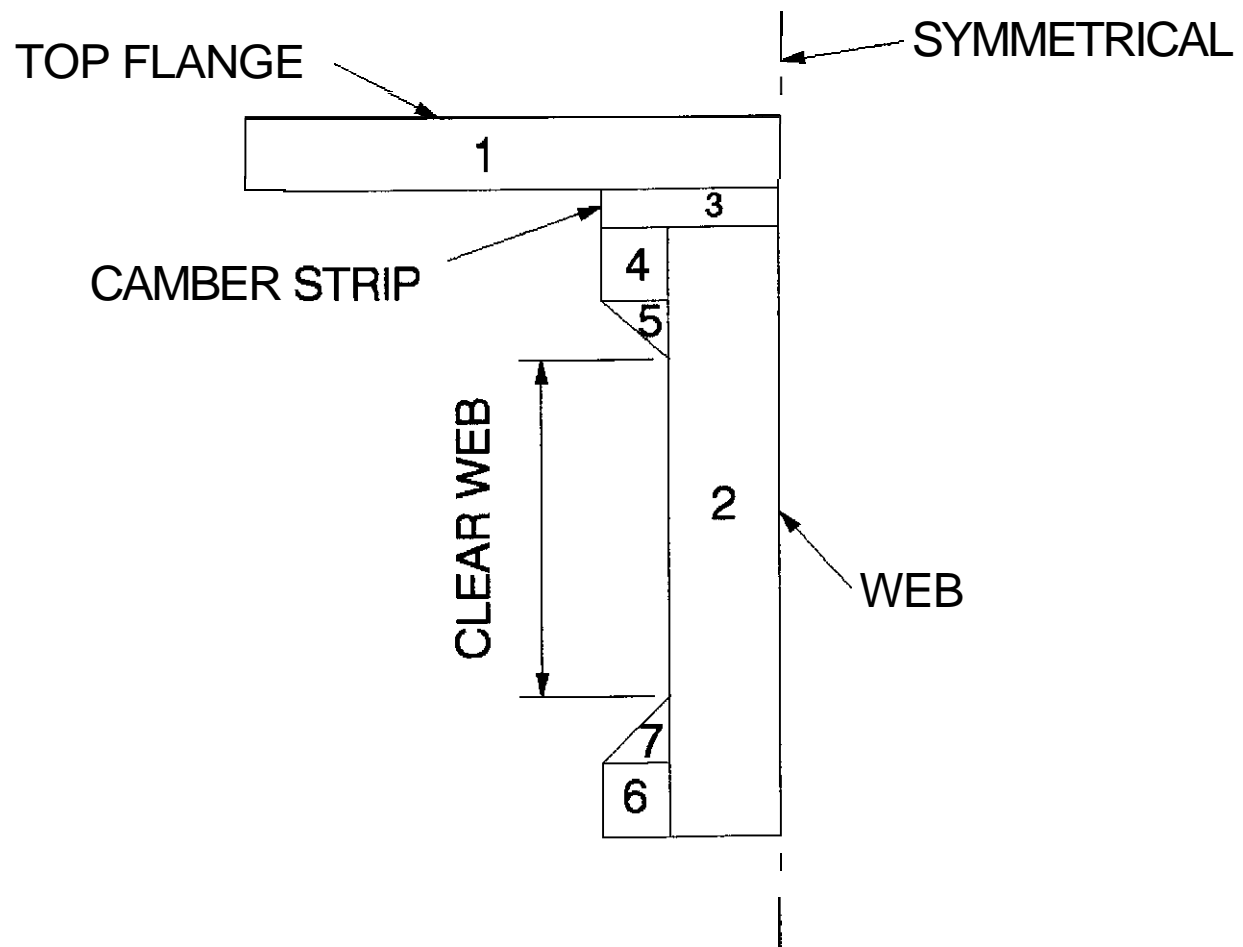


Figure 1 - Typical Cross Section for Input

number of areas in the half-section is 50. Area 1 must be the "top flange", and area 3 is reserved for a camber strip. If there is no camber strip (**e.g.**, when the entire section is precast), area 3 is assigned dimensions of zero. Area 2 must be a "**web**" area that extends from the bottom of the camber strip to the bottom of the section. More will be said below about this web area. Each area has seven attributes:

- (a) **WIDTH:** This attribute is the horizontal width of the area. For triangular areas, it is the horizontal base width. All areas must have at least one horizontal side. If the section has more than one web, use half the total width of all webs for the width of area 2. That is, area 2 represents half of the total "**web**" area.
 - (b) **HEIGHT:** Vertical height of the area.
 - (c) **TYPE:** 1 => rectangle; 2 => triangle (base on bottom); 3 => triangle (base on top)
 - (d) Distance from the top of the section to the top of the area.
 - (e) **CAST-IN-PLACE INDICATOR:** 1 => cast-in-place; 0 => precast. If the entire section is CIP, the section is to be input as precast.
 - (f) **CLASS:** 1 => above clear web; 2 => below clear web. This attribute is required to deal with the random variation of girder dimensions in the analysis program.
 - (g) Concrete compressive strength of the area.
4. The user enters the specific weight of the concrete, not including the weight of any imbedded reinforcing steel or other dead load.
 5. The user is prompted for the additional dead load that acts only on the precast section. This includes all dead load acting on the precast section alone, except the dead load of the precast concrete and any imbedded longitudinal mild steel or prestressing steel. If there is a cast-in-place deck, the dead load of the deck is not to be included, nor is any dead load that acts on the composite section.
 6. For design, the program requests the dead load that acts on the composite section.
 7. The number, locations, and area of rows of mild steel reinforcing bars are entered.

8. For analysis, the user is prompted for the number and locations of all prestressing steel (straight or harped rows, bundles, or ducts). Seven-wire 270 ksi strands are assumed.

DESIGN PROGRAM

The design program is named MXPSD (**M**ore **eX**act **P**re**S**tress Design). This program designs only for flexural stresses at **midspan** of simple span girders. The total number of required strands (with a specified center of gravity) is determined based on the full service load stress at the bottom of the girder at **midspan**. The design is based on **midspan** moments because the number of strands is governed by **midspan** stresses. Other normal design checks, such as at harp points and at girder ends, must still be made. Because MXPSD results in fewer strands, the stresses that usually govern at these other points will be lower.

Utilizing a very accurate analytical model, the program finds the required number of strands. The analytical model uses exact stress-strain curves for the concrete, mild steel, and prestressing steel. A detailed discussion of the model follows the description of the design and the analysis programs. Some important features of the design program are as follows:

1. The user is asked to specify the stress in the prestressing steel just prior to stress transfer. This should represent the initial stress minus any losses due to relaxation, shrinkage, friction, or anchorage that occur before stress transfer.
2. The user is prompted for the prestress losses that occur during and after stress transfer. Losses that occur before transfer, as explained in item 1 above, should not be included in these losses. Losses due to elastic shortening, creep, shrinkage, and relaxation that occur during and after stress transfer should be included.
3. The user selects the distance from the bottom of the girder to the **center-of-gravity** of the prestressing steel.

4. The minimum allowable compressive stress or the maximum allowable tensile stress to be allowed in the precompressed tensile zone must be specified.
5. The design program uses a number of assumptions for material properties:
 - (a) The mild steel modulus of elasticity is 29,000 ksi and its yield point is 60 **ksi**.
 - (b) The concrete modulus of elasticity is $33W^{3/2}(f'_c)^{1/2}$, where W is the weight of the concrete [lb/cu.ft.].
 - (c) The concrete flexural tensile strength is $6(f'_c)^{1/2}$. Note that this is not the design target stress in the precompressed tensile zone. See item 4 above.
 - (d) The prestressing steel ultimate stress is 281 ksi, the ultimate strain is 0.05, and the modulus of elasticity is 28,400 ksi. These values were taken from a study by Mirza et al. **(4)**.
6. The program output indicates the required number of prestressing strand. It also lists **midspan** concrete stresses at the top and bottom of the precast section under three different loading conditions: beam weight plus initial prestress; total dead load plus final prestress; and total load plus final prestress. If there is a cast-in-place deck, stresses at the top and bottom of the CIP concrete under full **service** loads are also given.
7. A minimum required concrete strength is calculated from the AASHTO allowables and the three load stages mentioned in 6 above.
8. The ultimate moment capacity provided by the section is calculated according to Article 9.17.3 of the 13th Edition of the AASHTO Bridge Code and compared with the required moment capacity.

ANALYSIS PROGRAM

The analysis program is named **PROCAT (PRObabilistic Capacity —Analysis Tool)**. The software will perform either a deterministic, mean-value analysis or a probabilistic analysis, both to check flexural cracking. The probabilistic analysis is accomplished by simulating a random sample, of specified sized, of prestressed girders. The program uses Monte **Carlo** simulation to produce the sample; a detailed account of how this is done is included in the discussion of the analytical model. The cracking moment capacity of each

girder in the sample is found using the same accurate analytical model used by the design program. A detailed discussion of the analytical model follows this section. The calculation of these moment capacities results in a corresponding sample (or population) of cracking moment capacities. The population of moment capacities, along with information about an extreme load, can be used to find a probability of failure (by flexural cracking) associated with the passage of the extreme load. Some important features of the analysis program are as follows:

1. The user has the option of either a deterministic, mean-value analysis or a probabilistic analysis.
2. For a probabilistic analysis, the user must enter two numbers to be used as the seed values for the random number generator. The sample size (maximum of 1000) must also be specified.
3. **The mean** prestressing steel stress just prior to stress transfer must be entered. See the discussion in item 1 for the Design Program above.
4. Because prestress losses are often a point of disagreement, the program offers maximum flexibility in this area. The user has the choice of specifying either a deterministic loss value (even though all other variables are randomly simulated) or the **mean** and **standard deviation** of the losses. For both options, default values are available. The default values are taken from a study by Mirza et al.(4).
5. For the deterministic analysis, the output consists of the mean cracking moment capacity of the girder and all of the input values for the girder (for user verification).
6. For the probabilistic analysis, the output consists of the mean and standard deviation of the cracking moment capacity. The input data are also included for user verification. In addition, the sample of cracking moment capacities is saved for statistical evaluation by the user.

ANALYTICAL MODEL FOR CRACKING MOMENT CAPACITY

The same analytical model is used in both the design and analysis programs. The model was first developed by Hognestad(2), was later used by Mirza et al.(4) and

TABLE 1

RANDOM VARIABLE RESISTANCE PARAMETERS

RANDOM VARIABLE	DIST.	MEAN	STD. DEV.	REF.
Top Flange Width	Normal	+5/32* in.	1/4 in.	[7]
Top Flange Depth	Normal	0.0* in.	3/16 in.	[7]
Web Thickness	Normal	0.0* in.	3/16 in.	[7]
Girder Depth	Normal	+1/8* in.	5/32 in.	[7]
Effective Depth to Mild Steel	Normal	+1/8* in.	11/32 in.	[7]
E of Mild Steel	Normal	29000 ksi	957 ksi	[6]
F _y of Mild Steel	Log Normal	66.8 ksi	5.5 ksi	[6]
Initial Prestress	Normal	189 ksi	2.8 ksi	[8]
Prestress Loss	Normal	36 ksi	5.75 ksi	[8]
Prestress Ulti- mate Stress	Normal	281 ksi	7.0 ksi	[8]
Prestress Ulti- mate Strain	Normal	0.05	0.0035	[8]
Prestress E	Normal	28400 ksi	568 ksi	[8]
Depth to Pre- stressing	Normal	0.0*	1/16 in.	[8]

*Deviation from nominal dimension.

Ellingwood et al.(5), and recently was applied to prestressed bulb-tee girders by **Geidl(3)**.

For the analysis program, if the deterministic analysis is selected, the properties used in the model are assigned mean values. For a probabilistic analysis, the random properties of a sample girder are produced by Monte **Carlo** simulation. The statistics of the random variables used in the analysis program are shown in Table 1. These values were taken from studies done by Mirza and **MacGregor(6,7)** and **Mirza et al.(8)**. The user can easily change or customize the statistical parameters if desired. For each random property, whether a material property or a dimension, a random number generator produces a random number from the appropriate distribution type. For example, if the random property is normally distributed, the random number generator used produces random values between negative and positive infinity; these random values have a mean of zero and a standard deviation of one and are normally distributed. The random number, in this example, represents a random number of standard deviations from the mean. The simulated random property is computed then by taking the mean value plus the product of the random number and the standard deviation of the property. For the design program, the properties are as described in the Design Program section.

The process of using the analytical model to find the cracking moment can be summarized as follows. Once the properties of a girder are determined, a curvature, ϕ , is imposed on the girder. The curvature represents the slope of the strain distribution on the girder cross section. The equilibrium moment that corresponds to that curvature must then be found. Since it **only** represents the slope of the strain distribution, by itself, the curvature is not sufficient information, to allow an equilibrium moment to be found. Therefore, as shown in Fig. 2, a trial value for the top-of-girder strain is also assumed in order to establish a trial strain distribution. For a girder with a cast-in-place deck, the

strain distribution is slightly more complicated as shown in Fig. 3. However, there is a fixed difference between the curvature of the precast girder and the curvature of the cast-in-place deck. Using the trial strain distribution and the stress-strain relationships for the materials, the total tensile and compressive forces, T and C , on the cross section are found. The stress-strain relationships used for concrete, prestressing steel, and mild steel reinforcing are taken from **Hognestad(2)** and **Mirza et al.(4)**. Fig. 4 and Fig. 5 show the stress-strain curves for concrete and prestressing steel, respectively. The mild steel is assumed to be elastic-perfectly plastic. If the net force ($T-C$) is not zero, a new trial value for the top-of-girder strain is selected and the process repeated. This process of varying the top-of-girder strain shifts the strain distribution as illustrated in Fig. 6. For example, if $T > C$, then the strain distribution should be shifted to the right to increase C and decrease T . If the net force is zero, the equilibrium moment that corresponds to the trial value of ϕ has been found. A schematic of the search for the equilibrium moment appears in Fig. 7. The equilibrium stress at the bottom of the girder is then compared to the flexural cracking strength of the concrete. If the equilibrium stress is not equal to the cracking stress, a new curvature is selected and the process is repeated; if the two stresses are equal, the equilibrium moment is the cracking moment capacity of the sample girder. An illustration of the search for the cracking moment is shown in Fig. 8.

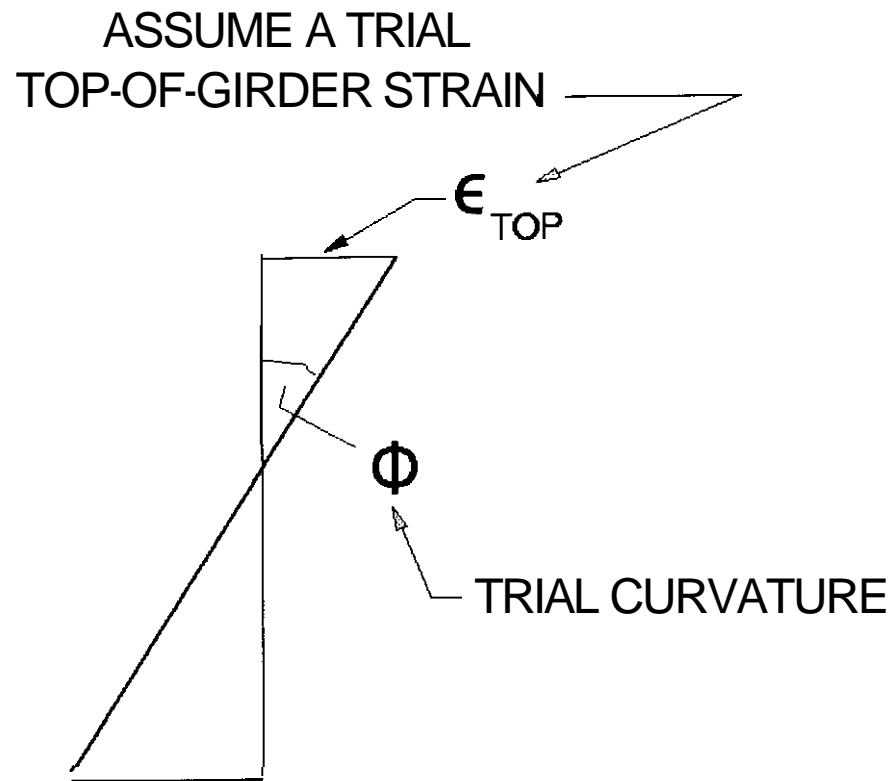


Figure 2 - Strain Distribution with Precast Deck

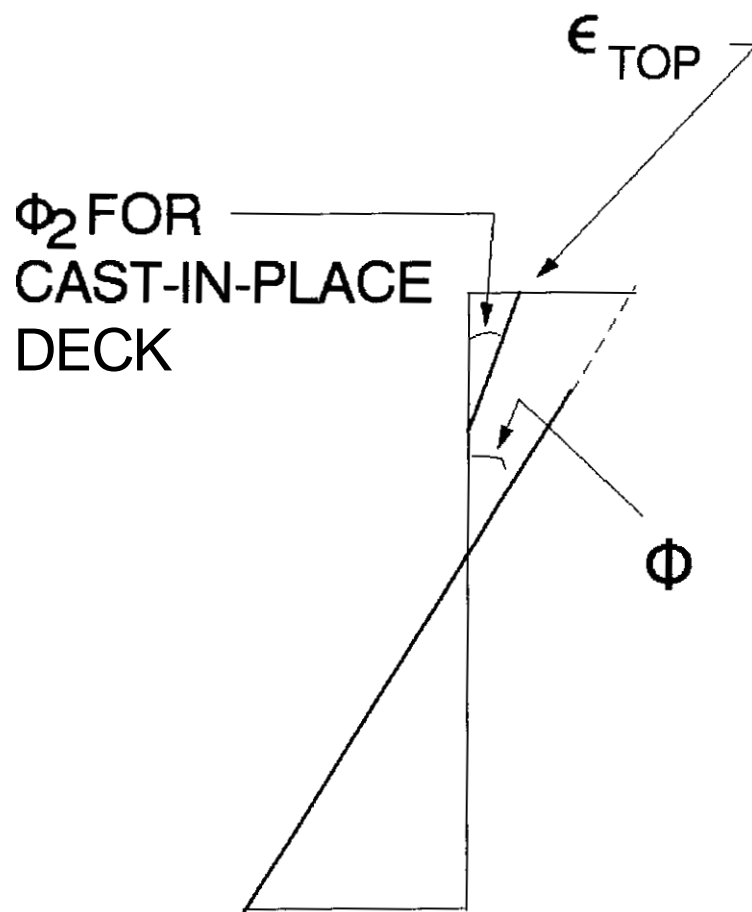


Figure 3 - Strain Distribution with CIP Deck

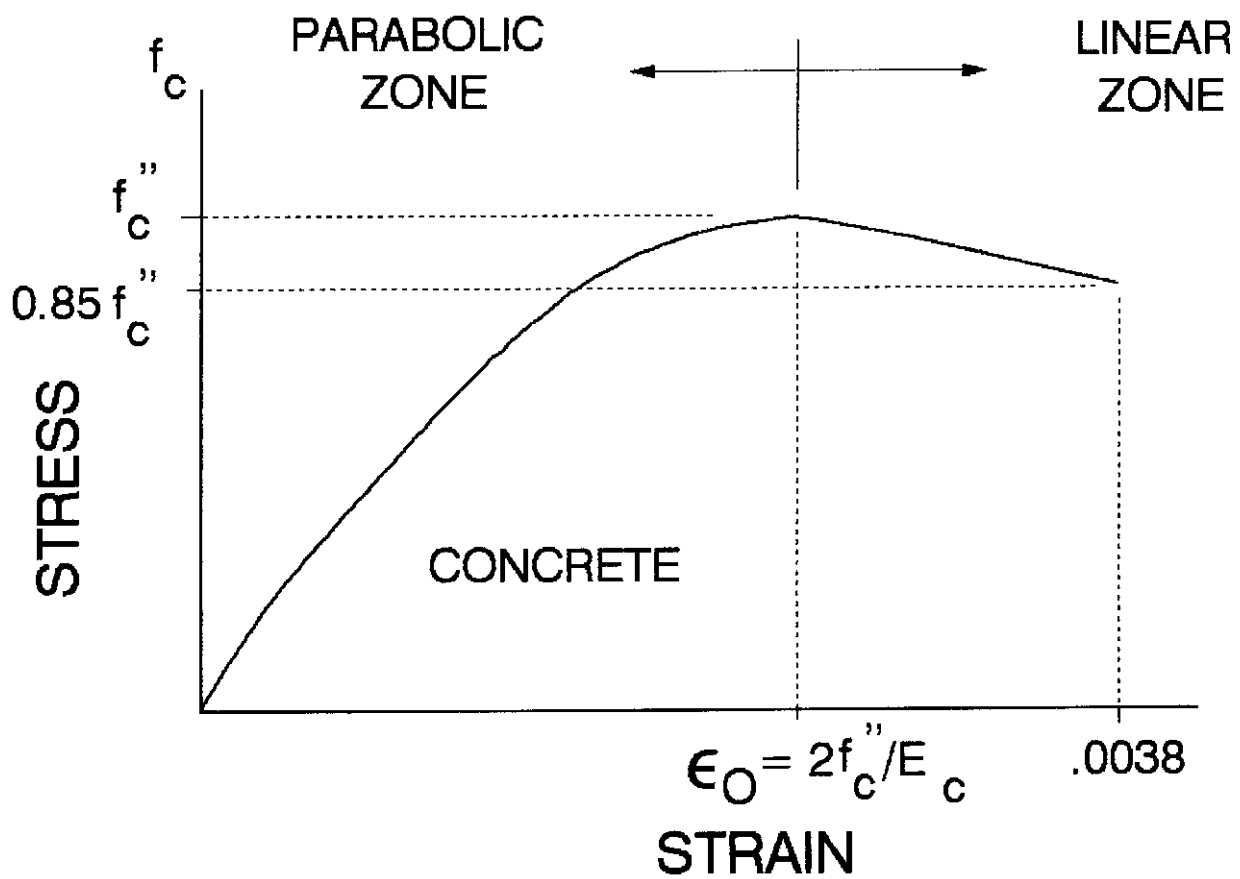


Figure 4 - Concrete Stress-Strain Curve

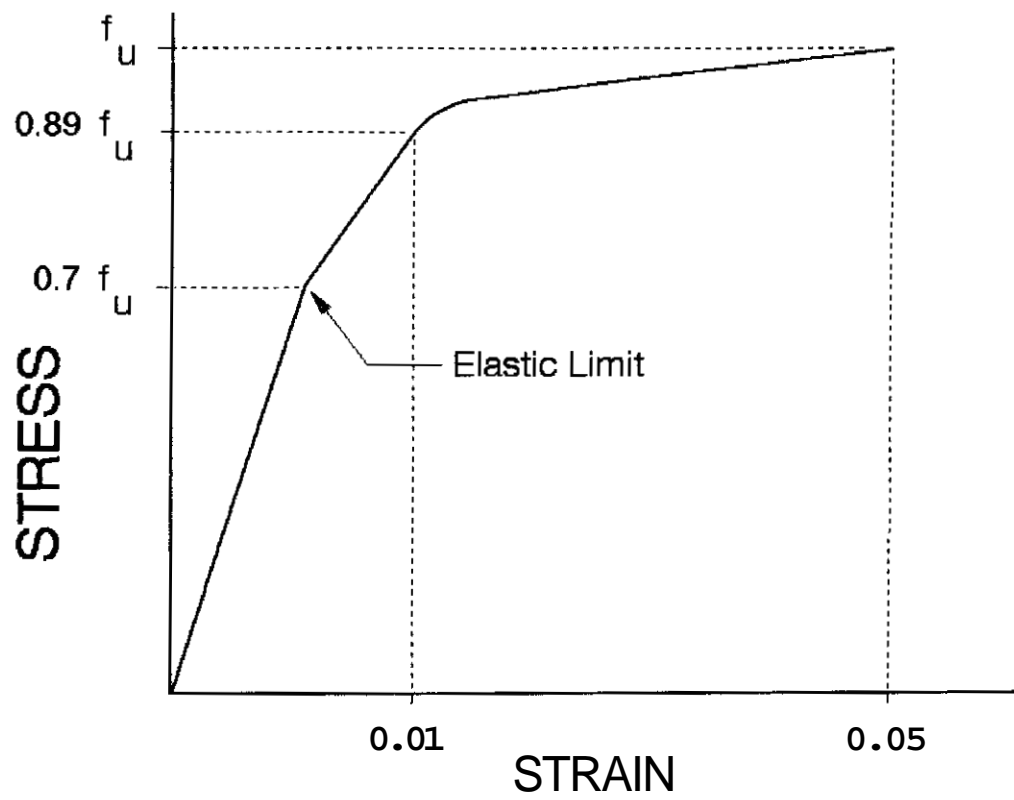
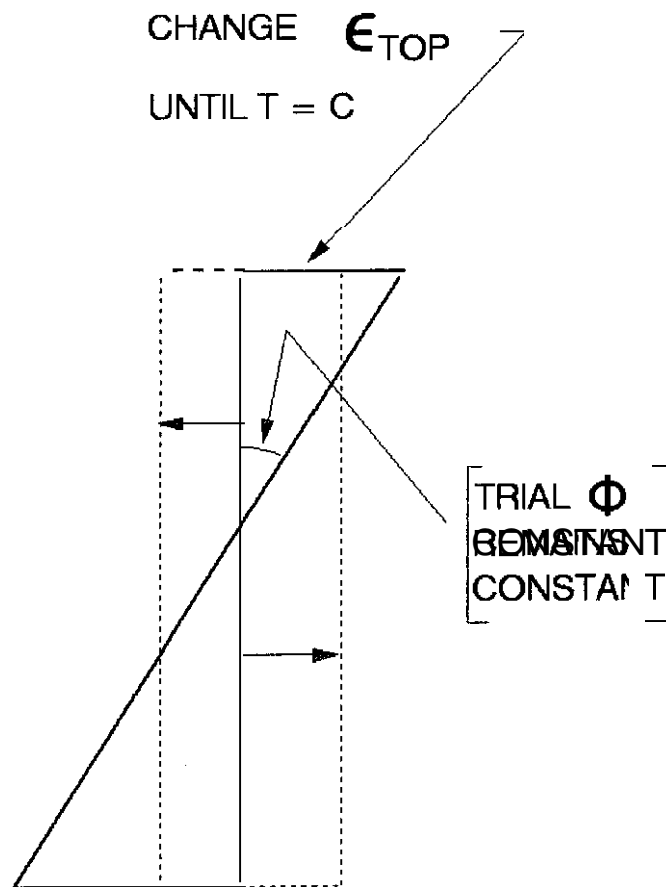


Figure 5 - Prestressing Steel Stress-Strain Curve



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Figure 6 - Shifting Strain Distribution

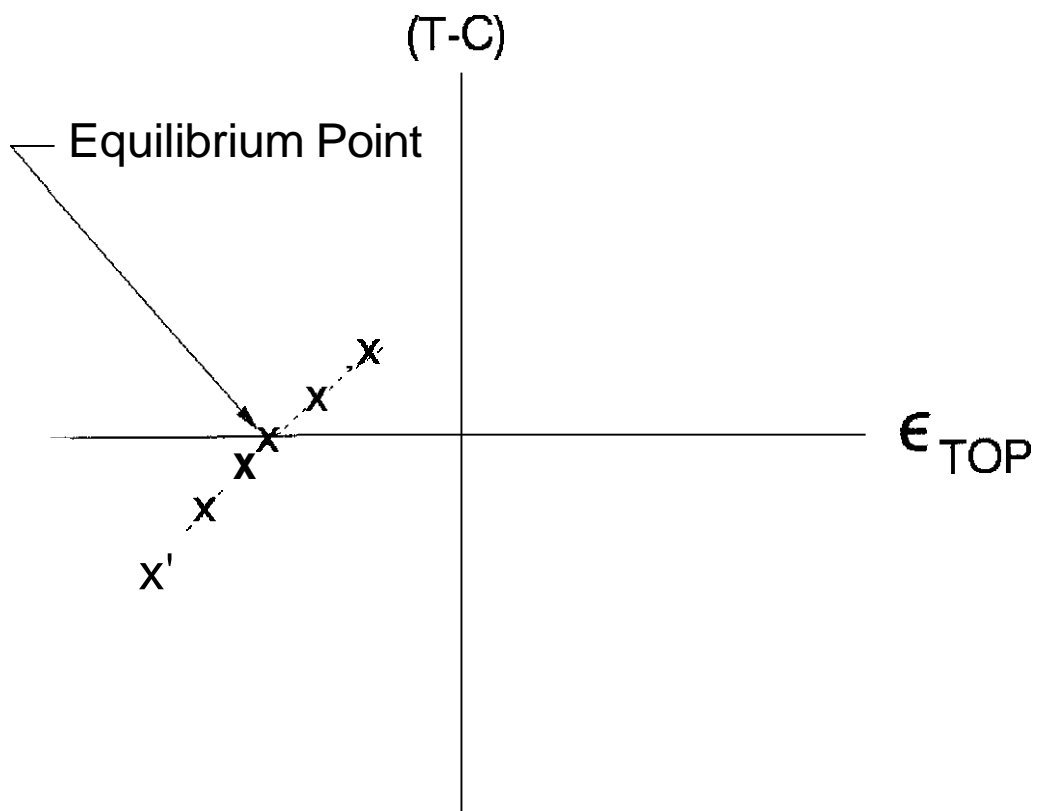


Figure 7 - Search for Equilibrium Moment

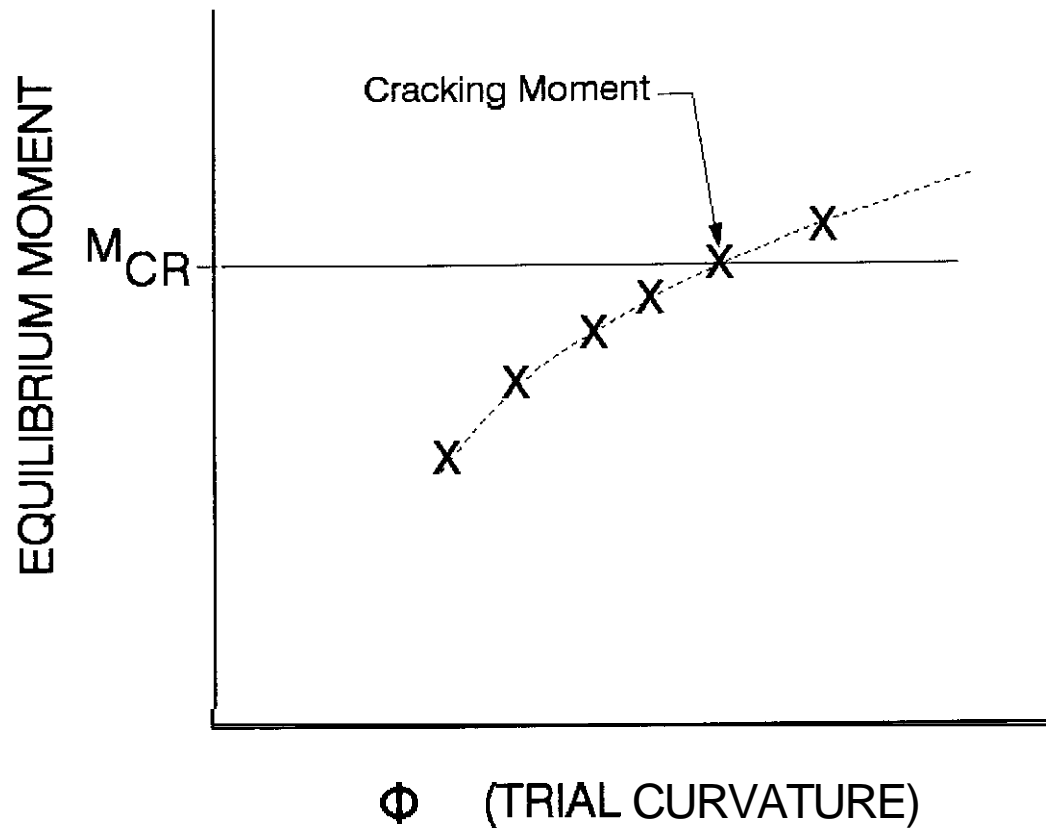


Figure 8 - Search for Cracking Moment

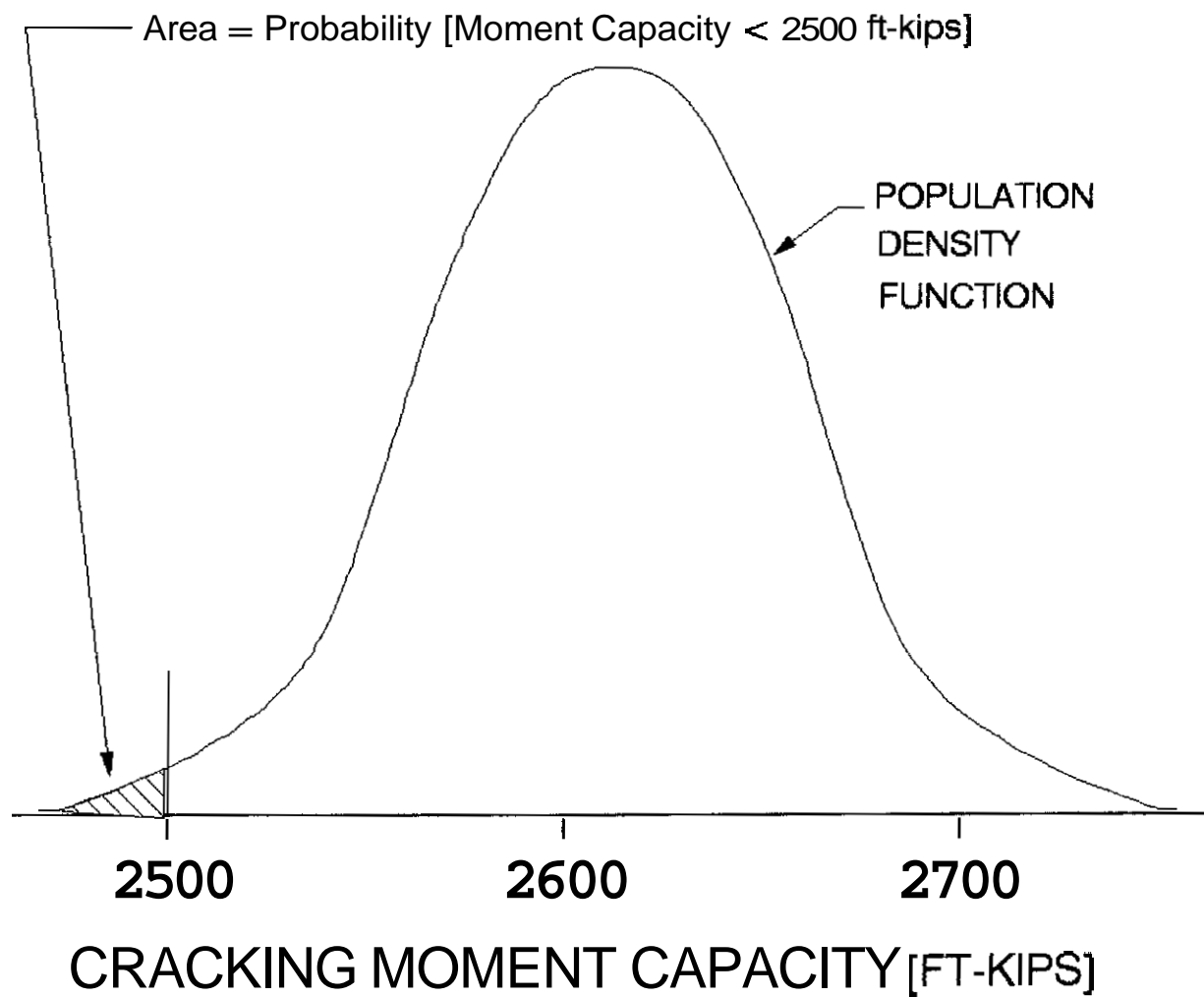


Figure 9 - Population of Cracking Moments

RESULTS AND CONCLUSIONS

DESIGN PROGRAM (MXPSD)

Results obtained by the design program were compared to several bulb-tee and **AASHTO** I-girder bridges designed by the Idaho Transportation Department (**ITD**). The spans varied from **85** to 109 feet. These results appear in Table 2. In general, the MXPSD designs required approximately ten percent less prestressing steel than the original designs. These results indicate that MXPSD can produce significant savings in prestressing steel for some bridges. The **ITD** generally uses a Prestressed Concrete Institute design program commonly called PSBRG. A number of theoretical designs were also done using PSBRG and compared to the results from MXPSD. The differences in the designs were similar to those shown in Table 2. The **ITD** uses a minimum concrete strength of **5000** psi in its designs, so no comparison could be made of required concrete strengths. However, since MXPSD generally results in lower concrete stresses, it is expected that additional savings could be realized where required concrete strengths are calculated.

ANALYSIS PROGRAM (PROCAT)

The probabilistic analysis produces a population of cracking moment resistances for a subject girder. Previous studies by **Geidl(9)** found that the population of moment resistances for bulb-tee girders could usually be represented by a normal distribution. No attempt is made in this report to recommend a parent distribution type for cracking moment resistances of prestressed bridge girders. For a given girder, the sample population produced by **PROCAT** should be statistically evaluated to determine an acceptable parent distribution for use in calculating a probability of failure. If a parent distribution can be

Table 2
Comparison of Design Results

PRELIMINARY COMPARISON OF DESIGN RESULTS		
BRIDGE	NUMBER OF STRANDS	
	ORIGINAL DESIGN	MXPSD DESIGN
COLBURN RD.	34	31
SNAKE R. BR.	38	34
QUINN RD.	44	39
EVERGREEN RD.	44	40

established, the probability of failure can be easily calculated using the cumulative distribution function. For example, suppose the population of moment resistances for a bridge girder turns out to be normally distributed with the parameters shown in Fig. 9. Further, suppose an extreme load needs to pass over the bridge and that the load will cause a moment of **2500 ft-kips** in the girder. Assume the extreme load is carefully controlled so that the **2500 ft-kip** moment can be taken to be deterministic (*i.e.*, not random). Using the mean, $\mu = 2610$, and the standard deviation, $\sigma = 35.6$, of the population, we first calculate the standard normal variable, z , and get

$$z = (2500 - \mu)/\sigma = -3.09$$

The probability that the girder will have a moment capacity, M , less than **2500 ft-kips** is

$$P[M \leq 2500 \text{ ft-kips}] = \Phi(z) = .001$$

where Φ is the standard normal cumulative distribution function. Physically, this probability represents the area in the tail of the population curve below a value of **2500 ft-kips** as shown in Fig. 9. This result indicates that there is a **1/1000** chance that the load would cause flexural cracking in the girder. An alternative to the use of a theoretical distribution, such as Φ above, is to use the actual, discrete cumulative distribution of the sample. Other methods are also available. In any case, using normal methods of reliability theory, a probability of failure can be calculated for either a deterministic extreme load or a probabilistic extreme load.

Some discussion of how to apply the results obtained by **PROCAT** is in order. The

statistical results that **PROCAT** provides are a valuable source of additional information. The engineer can use this additional information **to help** make a decision about the passage of the extreme load. That is, the information provided by using **PROCAT** should be not be seen as a substitute for engineering judgement, but rather as an additional aid to that judgement. The situation is similar to the way finite element techniques are used to help in this same area. The intention is that **PROCAT** will help the engineer to make better decisions about the safety of the passage of extreme loads. This should be very useful to agencies which deal with load limits, overload permits, or construction loads.

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